



Characterization of Friction in a Servo-Driven Test-Bed for Control and Simulation Purposes

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ABSTRACT

Precision control in certain critical systems, tracking systems with minimal error tolerance, and the availability of more tools of analysis, modelling and design. Friction is the tangential force between two contacting surfaces in relative motion. The surfaces in contact can be viewed from a microscopic point as contacting at a large number of points called asperities that are randomly distributed over the entire surface. The cumulative interactions of these bristles give rise to the friction phenomenon. In this paper, a series of experiments designed and implemented on a friction characterization and control test –bed is presented. The primary objective of friction characterization to the control engineer is for simulation and control design purposes. The results of the experiments show strong correspondence to other research findings as it relates the various friction features namely; pre-sliding hysteretic feature with non-local memory, frictional-lag, Stribeck effect and the stick-slip motion. This research findings and data will provide needed bases for system identification and modelling of system friction for precision control.

Keywords: Characterization of Friction; Control Systems; Pre-Slide Regime; Gross Slide Regime; Servo-Driven Test-Bed

Introduction

The study of friction and its associated phenomena cuts across various fields of research ranging from control engineering, tribology, geological sciences and meteorology. Tribological and control research findings have played a major role to the status of friction modelling and control. Tribology is the study of the science of contacting surfaces subjected to relative motion (Al-Bender and Moerlooze 2011), especially as it relates to friction, wear, and lubrication.

Friction has been described as the force between two contacting surfaces in relative motion. The surfaces in contact is viewed from a microscopic point as contacting at a large number of points called bristles or asperities that are randomly distributed over the entire surface. The cumulative interactions of these bristles give rise to friction. The presence of friction thus influences the overall behaviour of the surfaces under the effect of an external force (Miracle and Donaldson 2001). The theory of contacting surfaces gives rise to the distinction between the apparent surface area and the real surface area of contact. To the control engineer, understanding friction phenomenon is fundamental due to the increasing need for precision control in certain systems (Miracle and Donaldson, 2001; Yu and Adnan 2001; Zhang et al., 2006), tracking systems with minimal error tolerance, and the availability of more tools of analysis, modelling and design. Friction effects on mechanical systems are limit cycle oscillations, tracking errors and wear (Kok et al., 2001; Tijani & Rini, 2012). Two distinct friction regimes have been observed from research studies namely the pre-slide and the gross slide friction regimes. In the pre-sliding regime, friction is experienced in a system from the time of the application of an external force until the surfaces in contact are about to slide relative to each other. Here the friction is primarily a function of the displacement and is in opposition to the applied force. In this regime, the friction undergoes elastic and plastic deformation under the application of an external force. Increased application of force breaks the adhesive forces between the surfaces and initiates relative motion between them and initiates the gross sliding regime. The friction force seen when the contacting surfaces begin to experience relative motion because of the external force is a function of the velocity (Johnson and Lorenz, 1991 and Kemaio 2005). One of the primary aims in friction characterization is for the simulation and control purposes. In this paper therefore, a series of experiments were designed and implemented on a friction characterization and control test – bed to obtain such relevant friction features important to the control engineer. This is very important, as it is the basis for system identification of friction phenomenon and modelling. The results of the various experiments were compared with other research findings.

Features of Friction Relevant to Control

Adequate understanding of the characteristic features of friction phenomenon is paramount for its prediction, simulation and control. This is so because it forms the basis to gauge the adequateness of existing and future developments of friction models. Friction features are varied and non-linear in nature. Efficient system models are determined by their ability to replicate observed phenomena of such systems. Some of the more pronounced characteristics of friction to be discussed here are; stick-slip motion, the pre-sliding hysteresis, breakaway and varying breakaway forces, frictional lag, and the Stribeck effect.

Stick-slip Motion

Motions at velocities much lower than the Stribeck velocity often involve phases of slipping and sticking which can be traced to the stick-slip phenomenon of friction. As an illustration, consider a simple mechanical system consisting of a mass, spring system under the influence of friction as shown in figure 1. The application of a steady velocity in the direction indicated on the slider-belt will cause the spring extension while the mass is motionless relative to the slider-belt. This is called the stick-regime. The force transmitted to the spring is a function of the elongation of the spring (x), and the spring stiffness K . The mass remains motionless until the spring-force (Kx) is large enough to cause the attached mass to move backwards towards the fixed end of spring. This stage known as the slip-regime. Therefore, the mass experiences motion (slip) when the spring force (Kx) is greater than or equal to its static friction (F_s), at which point the mass begins to move and accelerates as a result of a fall from static friction to kinetic friction.

This acceleration is in the direction opposite the applied force, and hence reduces the stress in the spring. This causes the amount of force on the mass to decrease. The reduction of this force below a certain level causes the motion of

the mass to stop and stick again on the slider-belt. Hence, spring force begins to build up again to the static value replicating the whole process again, hence the name stick-slip motion. Figures 2 and 3 respectively capture the position of the mass as a function of time and the friction force as a function of time. This kind of motion is a feature of friction as observed experimentally and reported in some papers (Steffen et al 2012; Chen and Pan, 2000; Rabinowicz, 1958). Some examples are; the creaking sound of a door slowly closed or opened, motion of a wiper on a dry windscreen.

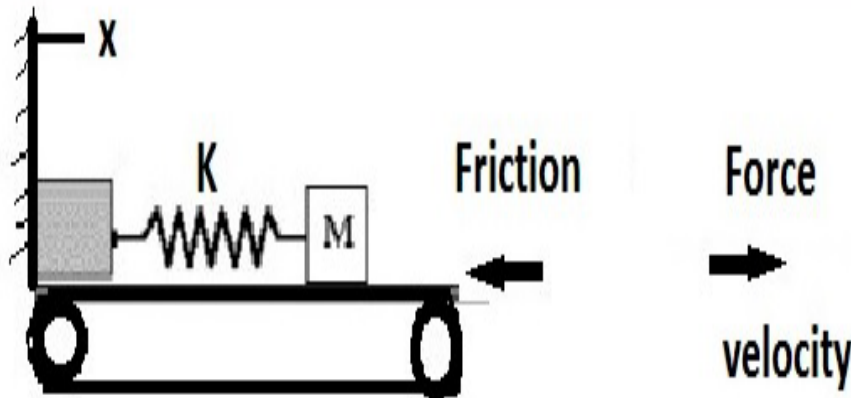


Fig.1 A simple block of mass-spring system subjected to a constant low velocity in the direction shown by the arrow used to demonstrate the stick slip motion

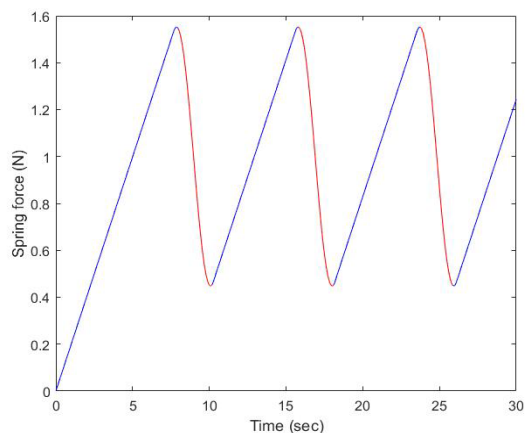


Fig.2 The motion of the mass as a function of time for the mass-spring system above showing the stick-slip oscillatory nature. The blue shows the sticking regime and the red the sliding regime

This type of motion is cyclic with periods of sticking and slipping as shown in figure 2 and the cause of many undesirable effects in mechanical systems like squeal noise and limit cycles

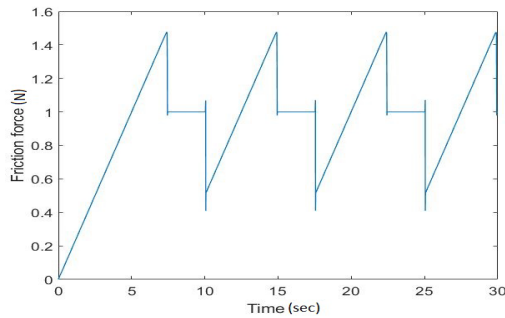


Fig.3 Friction force as a function of time for the system of figure 1. The maximum force is the static friction force and the flattened stage the kinetic (coulomb) friction force.

For the system shown the equation of the motion during stick as the body moves from rest is given by Newton's second law of motion

$$m\ddot{x} = F_s - Kx \dots \dots \dots 1$$

And that during slipping is

$$m\ddot{x} = F_c - Kx \dots \dots \dots 2$$

with $\dot{x} = v$, as the velocity, F_s and F_c are the static and kinetic friction forces respectively, K is the spring constant and x its extension as a function of time and m the mass of the body.

Hysteresis with non-local memory

This is the non-linear spring like behaviour of the asperities surface due to the interactions between forces of adhesion and deformation leading to the elasto-plastic deformation of these asperity junctions. This feature is a function of the displacement at micro-level rather than the velocity dependence of friction at macro-level.

This rate independent friction also exhibits a non-local memory feature. The relative displacement of the asperities is a function of the external force and increases to saturation beyond which the asperities slide (gross sliding) as a function of force until velocity reversal. Thus, the friction force increases in relation to this displacement until breakaway displacement beyond which the friction force becomes a function of velocity. Hysteresis losses occur at every closed cycle of operation as the stored energy is dissipated. This phenomenon is illustrated in figure 4 for a system subjected to a reference force input.

As an illustration of the non-local memory, consider a simple system as shown in figure 1, given the applied force of figure 4a, no motion is experienced between the mass and slider surface. This is possible when the applied force is less than the force necessary to break the adhesion forces of the asperities and initiate motion. Given the applied force (a – b – c – d – e – f – g), the asperities deform tracing first a virgin curve (a' – b') as shown in figure 4b, as the reference force changes direction moving from (b – c) the friction force traces (b' – c') curve. Reversal of the direction of applied force (c – d), the friction also traces the loop (c' – d') and as the force again reverses direction tracing (d – e), the friction force subsequently traces (d' – e') loop.

For the branch direction (e – f – g) of the applied force, the friction force traces the loop

(e' – f' – g'), however, as the friction force reaches the point f' coinciding with point d' on the outer loop (c' – d'), there is a closing of the inner loop (d' – e' – f') given that points d' and f' coincide. This thus makes the path (e' – f' – g') appear to trace the outer loop of c' – d' extrapolated, thus wiping out (or forgetting of) the inner loop (d' – e' – f'). This friction feature is termed hysteresis with non-local memory.

Frictional Lag

It was observed in (Bharat 1995), that the friction force and the velocity have a dynamic relationship at low but varying velocities. It was shown in the research that a periodic time varying unidirectional low velocity produces a corresponding friction force that lags the velocity in time. This friction force behaviour exhibits hysteretic features though a function of velocity as against that of the pre-sliding hysteresis, which is a function of displacement. Observations showed that the friction forces are higher for increasing time varying velocities (acceleration) than for decreasing time varying velocities (deceleration). Increasing the velocity time rate widens the hysteretic loop as in figure 5 for 2 rad/sec, 5 rad/sec, and 10 rad/sec sinusoidal reference velocities with constant amplitude.

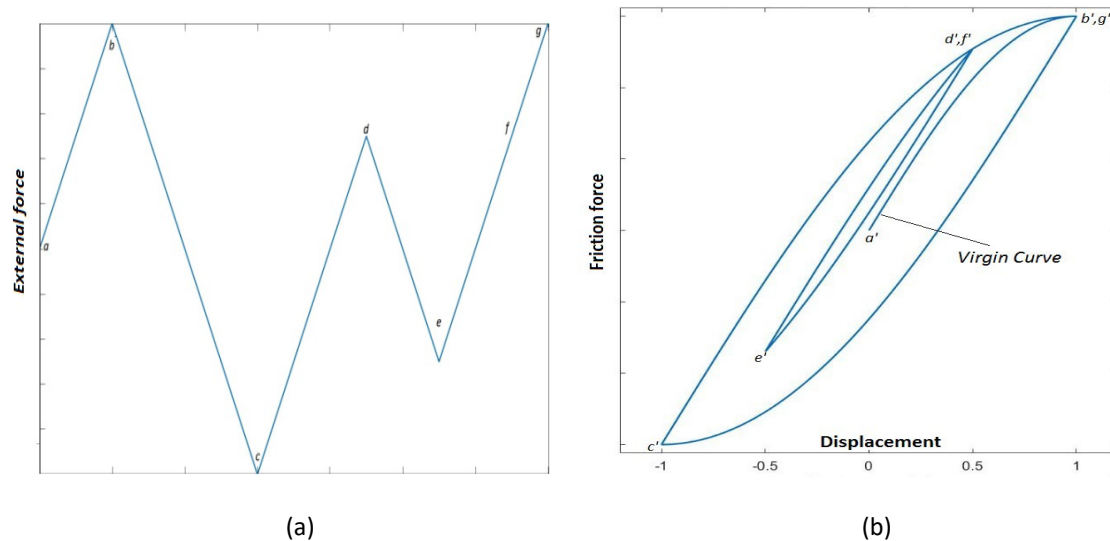


Fig.4 Non-local memory pre-sliding hysteresis;
(a) the reference force input for the pre-sliding hysteretic function investigation, and
(b) the pre-sliding hysteresis friction characteristic with non-local memory

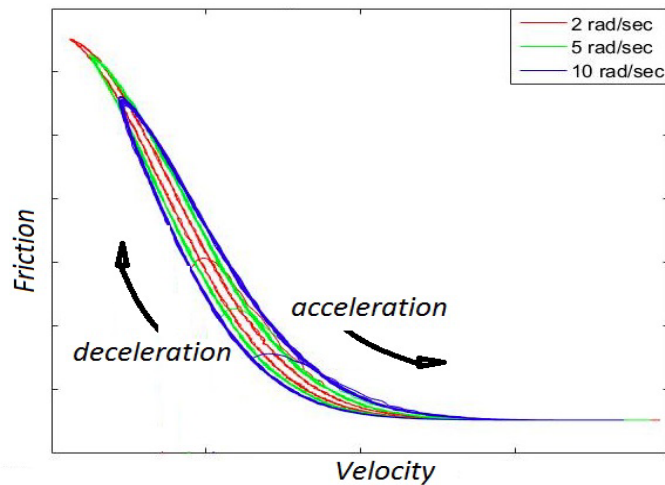


Fig.5 Frictional lag; friction as a function of unidirectional varying velocity input; the right arrow indicates increasing velocities (acceleration) and the left arrow decreasing velocities (deceleration).

Breakaway and Varying Breakaway Friction

The breakaway force is the minimum force required to initiate motion between two contacting surfaces. This force must be able to overcome the static forces of adhesion for a relative motion to begin. The transition from the pre-sliding (stick) to gross-sliding (slip) regime is dynamic and continuous. In his experiments (Steffen *et al* 2012), Rabinowicz noted that the maximum friction force occurs at a small displacement as shown in figure 6 and not at zero displacement indicating friction to be function of the displacement between the surfaces. The time of stick is seen to affect the stiction force in a manner that the longer the stick time the larger the stiction or the force required to initiate motion (figure 6a). The breakaway force is also found to be dependent on the rate at which the external force is changing, that is if the force rate is increased then the breakaway friction is decreased (McMillan 1997).

In other words, increasing the rate of application of force lowers the breakaway force and vice versa. Figures 6a, b, and c respectively show breakaway friction force, the breakaway friction force as a function of the rate of change of external force, and the breakaway friction force variation with total time of stick respectively.

Stribeck Effect

Prior to the experimental investigations carried out by Stribeck, friction has been thought of, and modelled as a static function proportional to velocity. In his paper (Johanness, *et al* 1973) and contrary to generally perceived notion, friction was demonstrated to be decreasing as velocity increased in the low velocity range while it increases in function of velocity beyond a certain level of the velocity. The transition velocity between the decreasing and increasing friction is called the Stribeck velocity. The friction-velocity relation called the Stribeck curve shown in figure 7.

The decreasing and subsequent increasing friction force as a function of the velocity increase is referred to as velocity weakening and strengthening respectively. The weakening is attributable to Stribeck effect and the strengthening to the viscous friction effect. Thus, the Stribeck friction is an exponentially decreasing function of the velocity bounded by the stiction force (upper bound) and the Coulomb force (lower bound) (Stribeck. and Wesentlichen 1902).

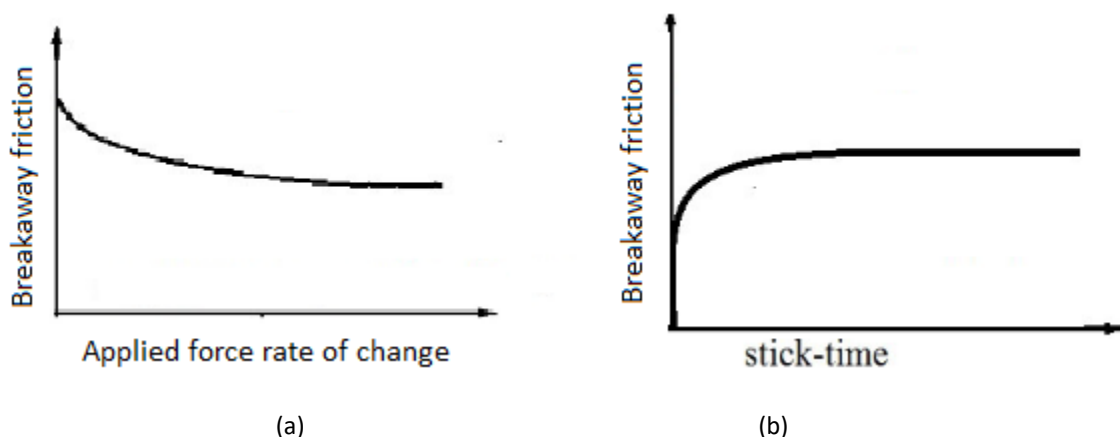


Fig. 6 Breakaway friction variation with: (a) displacement, (b) rate of change of force, and (c) total time sticking or dwell time

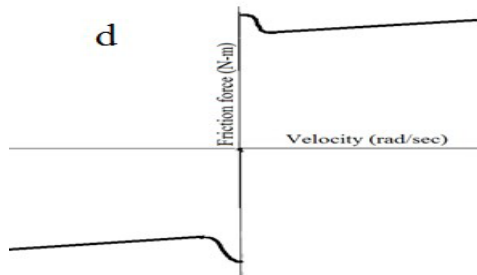


Fig.7. Friction as a function of velocity

The aforementioned friction characteristics will be experimentally investigated on an experimental test-bed designed for the purpose of friction characterization and control.

Methodology

The above-described friction features such as pre-sliding hysteresis, frictional-lag, breakaway variations make modelling friction a challenging task for the control engineer due to their nonlinear nature. Most of the existing models of friction are grouped as either static or dynamic depending on whether they are able to model friction dynamics as exhibited by real systems experiencing friction. Some of these models show greater representation of friction than others though generally at the expense of some other relevant factors such as computation complexity, simulation efficiency. The purpose in this section is to investigate through experiments these described friction characteristics on an experimental test-bed.

To achieve this, experiments were designed and performed using an experimental friction

test-bed specifically designed and constructed to test and examine friction characteristics. The experiments performed reflect the various features of friction as it is impractical to design a single experiment capable of demonstrating all relevant phenomena of friction.

The constant-velocity experiments is performed to capture the Stribeck effect, which is an important characteristic of the friction phenomena. The second experiment is the frictional-lag experiment designed to show that friction for increasing unidirectional velocity signals is larger than that for decreasing unidirectional velocity signals. Friction-displacement experiment is performed in the pre-sliding regime to obtain the hysteretic features of friction with non-local memory characteristics against the previously held opinion of non-memory-based hysteresis.

Experimental Test-Bed

An experimental test-bed for friction characterization and control was developed in the control laboratory with the SRV-02 as the base plant at the University of Reading, United Kingdom. The diagram representing a picture of the experimental test-bed is presented in figure 8, and it consists of many components such as the torque sensor, friction load discs, the SRV-02 rotary servo system housing the motor, tachometer and the encoder. The SRV-02 rotary servo from Quanser is powered by a dc power supply providing needed DC to drive the motor, the angular speed of the motor shaft is measured by the tachometer attached to the motor shaft, and the encoder measures the angular position of the load disc in a quadrature mode. The friction load discs, made from mild-steel, aluminum and copper. The friction torque between the static and moving discs are measured using the torque sensor coupled to the static disc. A data acquisition card (Q2-USB) was used to interface the hardware (SRV- 02) with the Quanser Quarc software running on a PC. This software is accessed using the MATLAB/SIMULINK software tool to run the real time experiment in a Hardware In the Loop (HIL). Manner. The friction discs, torque sensor and the motor shaft are aligned properly to ensure a uniform distribution of normal load over the entire surface. Friction is a result of the relative motion of the motor shaft and the static load disc and is measured by a torque sensor connected to the load disc. The torque sensor is powered through a 14v dc power supply and amplified by the CSG110 amplifier to provide the

right excitation to the torque sensor. Various experiments were then carried out using different input references in the SIMULINK environment as would be described subsequently.

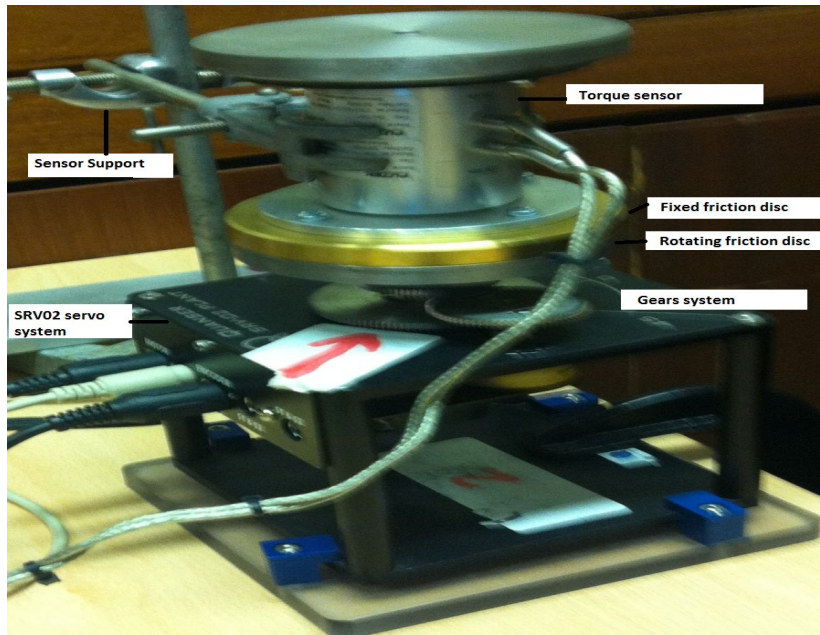


Fig. 8 A picture of the friction test-rig

Experimental Procedures

The series of experiments performed on the friction experimental test-bed are designed to capture relevant friction phenomena such as the Stribeck effect, the frictional lag phenomenon, the pre-sliding friction hysteresis with non-local memory. The nature of the input-output data set was determined prior to the experiments in order to capture the characteristics of interest.

Experiment 1: Friction-constant velocities characterization

Objective: Determination of friction as a function of velocity

Background: Some research findings pointed out earlier indicate that at very low velocity ranges the friction curve has a negative slope for velocity increments. However, beyond a velocity threshold called the Stribeck velocity (v_s), the friction-velocity relation becomes more linear and positively sloped. This behaviour is thus termed the Stribeck effect.

Procedure: For each reference velocity input the system was run for 15 seconds and the output velocity measured using the tachometer at a rate of 1ms obtaining a total 15,000 data points. The first 3,000 and the last 1,500 of these measured data were discarded and the average of the remaining obtained used. This is done to ensure the elimination of transient behaviour thus allowing the system settle close to its true value since for this experiment the interest is the steady state values. The value of the friction torque was also measured by the torque sensor and recorded for the same data points as for the measured velocity. For each velocity reference, the experiment was performed 10 times and the mean value recorded as the final friction torque and velocity data points. This is done to eliminate errors and thus increase the accuracy of the data in the light of its true value and it was observed that 10 runs were adequate for the purpose. The reference velocities for the steady-state experiment range from very low values of 0 rad/sec-to 1 rad/sec. Thus, a total of 80 friction torque-velocity data points was obtained. Figure 9 shows the results.

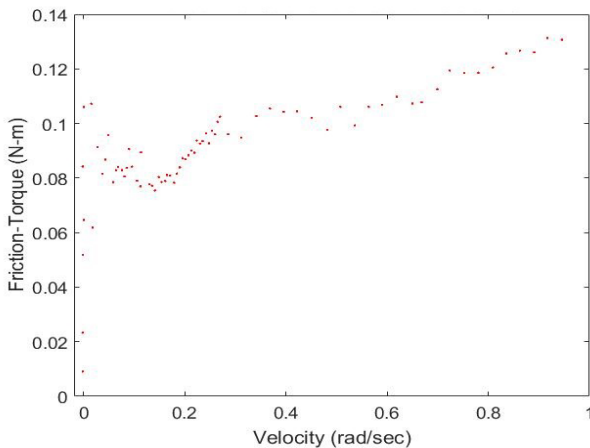


Fig. 9 The measured system friction plotted against velocity

Experiment 2: Frictional- Lag Characterization

Objective: To determine the frictional torque variation as a function of varying unidirectional velocities.

Background: There is a relative time-lag between the friction torque and the corresponding velocity, in the sense that the system-friction does not respond instantaneously to system inputs. This lag gives rise to a hysteresis effect in the velocity regime similar to the hysteresis effect in the pre-sliding displacement regime. Research shows that the frictional torque is larger for increasing velocities (acceleration) than for decreasing velocities (deceleration), the loop of the frictional lag encloses the Stribeck low velocity curve indicative of the vanishing of the former for increasing velocities. Thus, the constant velocity-friction torque curve at low velocity acts as an attractor to both the acceleration and deceleration friction torques.

Procedure: In designing the experiments, a time varying periodic and unidirectional reference signal is used. To obtain this a pure sinusoidal signal was superimposed upon a constant positive signal whose value is greater than the amplitude of the sine sinusoidal signal. This is to ensure adequate capture of low velocity variations as a function of time without the signal swinging between positive and negative values. To achieve this a sinusoidal velocity signal of the form is used

$$v(t) = A - B \sin(\omega t) \dots \dots \dots .3$$

Where:

A is a positive bias velocity signal chosen in such a way as to ensure the velocity is always unidirectional i.e. B is less than A . the values of omega: $\omega = 1$ rad/sec.

Constraint: The variables B and A have to be chosen so as to make the total velocity unidirectional in order to capture the Stribeck slope as suggested in the experiment 1. They were chosen to ensure positiveness of the time varying velocity at all times. So $A = 1$ and $B = 0.95$ and as such the periodic velocity becomes:

$$v(t) = 1 - 0.95 \sin(\omega t) \dots \dots \dots .4$$

The choice of these values were such that true sliding is ensured avoiding periods of sticking and zero or reversal velocities. For each run of the experiment, the corresponding values of the friction-torque and velocity is obtained for the acceleration and deceleration regimes. The experiments were performed as in experiment 1 above and the average of the measured friction-torque and velocity recorded. The results so obtained is shown as figure 10.

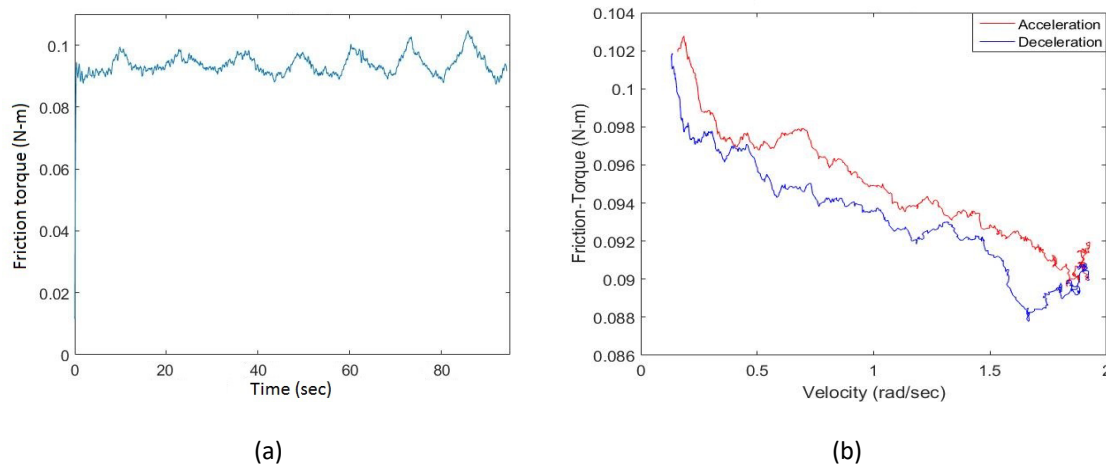


Fig. 10 The Frictional-Lag experiment: (a) Friction against-time, and (b) The plot of friction variations for increasing (acceleration) and decreasing (deceleration) unidirectional velocity regimes

Experiment 3: Pre-Sliding Hysteresis Characteristics

Objective: To determine the hysteretic relationship between the friction and displacement in the pre-slide regime.

Background: As stated previously, the pre-sliding hysteresis with non-local memory has been established as against the non-memory-based behaviour earlier believed to govern the pre-sliding regime. This characteristic is independent of the velocity of the bodies in contact indicative of the pre-dominance of the elastic deformation characterizing the bristles behaviour.

Procedure: The experiment to determine the pre-slide friction hysteresis is carried out in two stages as follows;

Stage 1: A ramp torque signal was injected into the system slowly incrementing it till the time of gross-sliding and the surfaces moving relative to each other. This process is repeated several times and the average friction torque and the applied torque input were recorded thus giving us a range of values for which the system is in the pre-slide regime and that beyond which a relative motion was observed. This breakaway friction is also called the Stiction. The value of the pre-sliding displacement is also recorded to determine a range of values for the pre-sliding regime before breakaway. By this, the breakaway friction and the corresponding breakaway displacement were obtained. In the same way, a negative signal is used to determine the range for the negative breakaway torque and the corresponding breakaway displacement.

Stage 2: A special displacement input signal shown figure 11a, is then used to ensure the Stiction torque range as earlier determined in stage 1 above was never exceeded both in the negative and positive going reference signal inputs. The measured friction torque from the experiments is shown in figure 11b. This experiment was done in same manner as in experiment stage1 and their average values for friction shown in the figure.

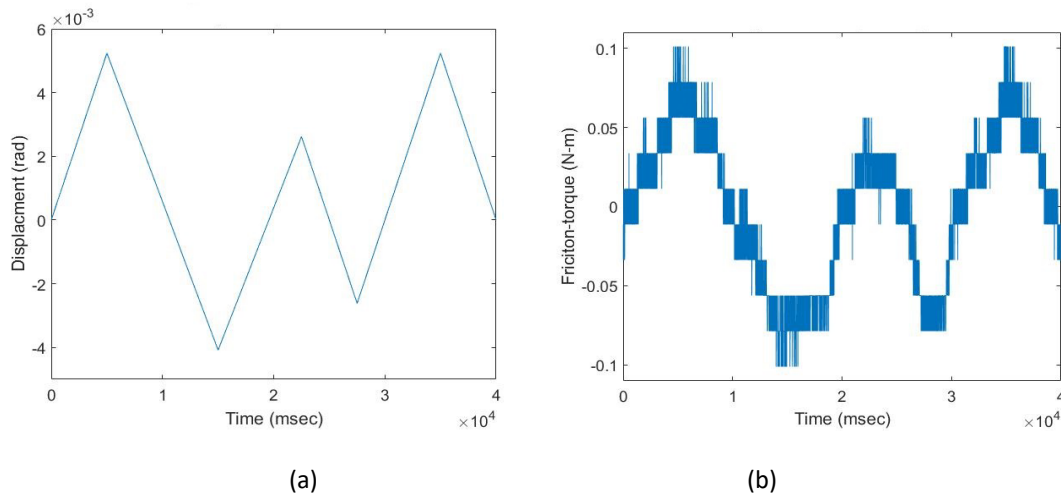


Fig. 11 The Pre-sliding Hysteresis with non-local memory (a) Reference input signal, and (b) Measured hysteretic friction-torque output showing quantization

Analysis of Results

Here an analysis of the various results obtained from the experimental test-bed designed to study friction behaviour in the laboratory is presented. The presentation is structured similar to the previous section for the various characterization experiments.

Constant-Velocity Characterization Results

During the run of the experiments, it was observed that as the velocity is incremented from zero with a step of 0.03 rad/sec the test-bed showed no relative motion, the offset error due to the tachometer was corrected. Increasing the input signal the measure friction output was observed to increase, though the output velocity remained zero indicating static friction build up. From a velocity value of 0.1 rad/sec, the system was seen to begin to show momentary motions marked with periods of elongated slip followed by small stick. This region was characterized with moments of slip and stick motions. The system continued with this motion pattern with increasing slip moments and decreasing stick moments. This thus marked the transition range from sticking regime to the gross-sliding regime was shown to be a range of values rather than single valued. The moment the disc began to exhibit increased slip the torque sensor reading indicated a fall in the values of the measured torque between the surfaces after which it tends to be more constant. This behaviour is indicative of the fact that for the system the Stiction (static) friction is greater than the coulomb (kinetic) friction. This torque at the point of slip is the Stiction or the breakaway torque and the friction at constant motion the coulomb-viscous friction. From the plot of the friction-torque versus velocity shown in figure 9, it is obvious that at low velocities (between 0 and 0.2 rad/sec) the friction-torque generally decreases as the velocity increased typical of the Stribeck effect of friction at low velocities. Research findings support this important feature of friction as earlier highlighted in the literature review. From the figure, in the region of low velocities one notes that the friction velocity measurements are rather erratic with much variation but generally showing a negative slope in an exponential manner. This relative high degree of variation in the low velocity underscores the fact that friction is very difficult to effectively capture and model in this range of velocities. This could be due to forces of adhesion, and non-uniform asperity deformations, whether the net bristle distributions are in trough (valley) or on mountain contact scenario.

Other possible factors contributing to these large variations are system noise in the measuring devices like the torque sensor and environmental factors such as temperature and lubrication. From the graph of figure, the medium to high velocity range (from 0.2 and above) there seemed to be a more linear relationship between the friction-torque and the velocity as the measured torque steadily increased as velocity increased. This behaviour is also evidently

supported by many research findings. In this medium to high velocities range, the friction torque could easily be modelled using classical models of the coulomb-viscous model with minimal errors. This behaviour of the friction phenomenon as a function of increasing velocities from zero is thus called the Stribeck effect and the graph of the relationship called the Stribeck curve.

Varying Velocity Characterization Results

Close examination of the frictional-lag features of figure 10b suggests some erratic behaviour (for velocities much lower than the Stribeck velocity) whereby the friction torque at a particular velocity instant is seen to be lower in the acceleration regime than the deceleration regime contrary to the general known friction behaviour. Generally, results of figure 10b for the varying velocity characterization experiment showed a correspondence with research findings, (Steffen 2012, Al-Bender & Moerlooze 2011). In the experiment, it was observed that the friction torque corresponding to a given velocity does not happen instantaneously but after some lag in time and as such the torques for acceleration periods were higher than those for deceleration. The width of the lag was seen to widen as the reference input velocity increased. In other to relate these inputs were varied to 2 rad/sec, 5 rad/sec, and 10 rad/sec. The results of these effects was rather made more pronounced using the friction model to plot the friction against the various velocities as shown in figure 5 above.

Pre-Sliding Hysteresis Characterization Results

The graph for the pre-sliding experiment showed the non-local memory characteristic of this regime as explained earlier. This dynamic behaviour suggests friction to be displacement dependent in the pre-slide regime and independent of velocity of motion between the surfaces. From the results of the pre-sliding experiments performed figure 11b, the following observations could be made:

1. The asymmetric nature of friction such that the negative velocity regime and the positive velocity regime each give rise to different parameter values like Stiction and Coulomb forces.
2. That friction is mostly non-linear near zero velocities and at zero velocities.
3. That friction is proportional to displacement in the pre-sliding regime and may not necessarily be zero at zero displacement. As a result of the above, it is not easy to capture the rich dynamics of the pre-sliding regime of the friction phenomenon.

Worthy of note is the observation that the friction as measured by the torque sensor appear to be quantized especially in the pre-sliding regime. This is primarily due to the measured torque values being very small in this regime, when compared to the capacity of the torque sensor, which is about 23N –m.

Conclusion

Presented in this paper are various experiments relevant for the characterization of the friction phenomenon. This is an important step in system identification for the modelling and control of friction in systems. As friction cannot be completely eliminated in contacting surfaces experiencing relative motion, its compensation becomes necessary. The results of the experiments show strong correspondence to other research findings as it relates the various friction features namely; pre-sliding hysteretic feature with non-local memory, frictional-lag, Stribeck effect and the stick-slip motion. The data so obtained from the various experiments will be used in further research such as performing friction modelling and system identification, and friction compensation and control design for systems with friction such as the test-bed used for this experiment.

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